

## Two problems in thermal field theory\*

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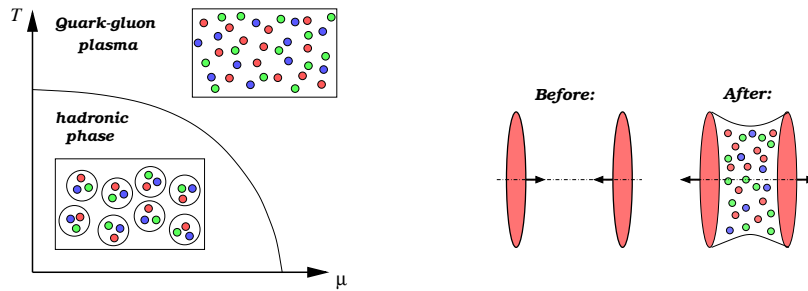
**Abstract.** In this talk, I review recent progress made in two areas of thermal field theory. In particular, I discuss various approaches for the calculation of the quark gluon plasma thermodynamical properties, and the problem of its photon production rate.

**Keywords.** thermal field theory, quark-gluon plasma

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### 1. Introduction

At low temperature and density, quarks and gluons appear only as constituents of hadrons because of confinement. However, QCD lattice simulations predict that above a certain temperature or density, nuclear matter could undergo a phase transition after which quarks and gluons are deconfined from hadrons, and form a new state of matter called “quark-gluon plasma”. In order to create in the laboratory the conditions of temperature and



**Figure 1.** Left: simplified phase diagram for QCD. Right: a heavy ion collision.

density necessary for this transition, heavy nuclei are collided at very high energies. To be

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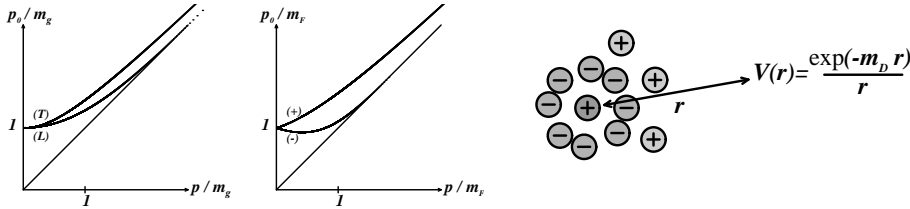
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able to detect the formation of the quark-gluon plasma in such collisions, one needs to find observables that are sufficiently different in a QGP and in hot hadronic matter.

Thermal field theory is one of the possible tools for studying the plasma phase. Basically, thermal field theory is the formalism obtained by merging quantum field theory and the tools of statistical mechanics: its only difference with zero temperature field theory is that it incorporates a statistical ensemble of particles in the system, and that those particles can participate in reactions. More formally, the statistical ensemble appears in the definition of thermal Green's functions: at statistical equilibrium, the thermal average of an operator  $A$  is defined as  $\text{Tr}(e^{-H/T} A)/Z$  instead of  $\langle 0|A|0\rangle$ . In order to calculate perturbatively these thermal Green's functions, there is a set of Feynman rules very similar to the zero temperature ones [1].

An improvement over the bare thermal perturbative expansion has been proposed in the early nineties by [2], which amounts to the resummation of 1-loop thermal corrections. Indeed, it was realized that these loop corrections (known as Hard Thermal Loops) are of the same order of magnitude as the corresponding tree-level amplitude when the external momenta are soft. In this context, *hard* refers to momenta of order the temperature  $T$ , while *soft* means of order  $gT$ , where  $g$  is the coupling constant of the theory. The resummation of HTLs can be seen as a reordering of the perturbative expansion.

Physically, this resummation modifies the dispersion curve of elementary excitations of the theory (see Fig. 2 for QCD) by providing them a thermal mass of order  $gT$ . Additionally, in the static limit ( $p_0 \rightarrow 0$ ) in the space-like region, the longitudinal gauge boson has a non vanishing self-energy  $m_D^2 \sim g^2 T^2$  which provides a screening of static electric fields in the plasma. This phenomenon is known as Debye screening (see the illustration on



**Figure 2.** Left: gluon dispersion curves. Middle: quark dispersion curves ( $m_g$  and  $m_F$  are thermal masses of order  $gT$ ). Right: Debye screening of the electric field.

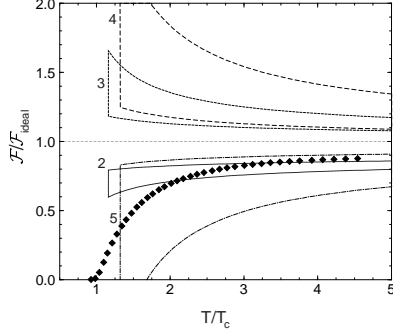
Fig. 2) and is due to the fact that a test charge placed in the plasma polarizes the medium around it, so that the Coulomb field it creates at large distance is exponentially suppressed. At this level of approximation, there is no screening for static magnetic fields<sup>1</sup>. Finally, the resummation of hard thermal loops also includes in the effective theory the Landau damping, coming from the fact that the quark and gluon self energies have an imaginary part in the space-like region. Note also that these self-energies are purely real in the time-like region, which means that the elementary excitations are stable in this framework. Their decay is a next-to-leading effect, and their lifetime is of order  $(g^2 T)^{-1}$ .

<sup>1</sup>In QED, this result is known to hold at all orders of perturbation theory. However, this proof cannot be generalized to QCD. In fact, a magnetic screening mass is expected for QCD at the non-perturbative scale  $g^2 T$ .

## 2. Thermodynamics of a quark-gluon plasma

### 2.1 Convergence of the perturbative expansion

A lot of work has been devoted in the past years to the calculation of thermodynamical properties of a quark gluon plasma, and in particular of its pressure. The pressure can be obtained by differentiation with respect to the volume from the free energy, which is itself given by  $F \equiv \ln \text{Tr}(\exp(-H/T))$ .  $F$  can be calculated perturbatively as a sum of vacuum diagrams. Although this problem looks straightforward, early attempts showed that the bare perturbative expansion of this quantity has a very poor convergence [3,4]. Practically, the coefficients in front of higher orders in  $g$  were found to be quite large,



**Figure 3.** Normalized QCD (without quarks) free energy as a function of temperature [3]. The dotted line is a result from lattice QCD [5]. The various curves come from perturbation theory respectively to order  $g^2$ ,  $g^3$ ,  $g^4$ , and  $g^5$ . The bands correspond to a variation of the renormalization scale by a factor 2.  $T_c$  is the critical temperature.

limiting the applicability of these results to very small values of the coupling constant. This is illustrated for QCD on Fig. 3.

### 2.2 Screened perturbation theory in scalar theories

In order to overcome this difficulty, various resummation schemes have been proposed to improve the convergence of the perturbative expansion. Let me first discuss some of those strategies in the case of a scalar field theory with a quartic coupling  $g^2\phi^4$ , since this model illustrates the methods while avoiding complications related to gauge invariance. In this model also, the convergence of the bare perturbative expansion is very poor [6].

A first method used to solve this problem is known as “screened perturbation theory”, and amounts to reorganize the perturbative expansion by inserting a fictitious mass term  $m$  in the Lagrangian [7]:

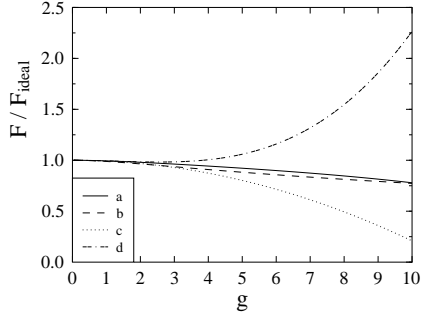
$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \frac{g^2}{4!}\phi^4 + \frac{1}{2}m^2\phi^2. \quad (1)$$

The idea behind this improvement is that medium effects tend to give a mass to elementary excitations: in other words, some of the interactions can be hidden in an effective mass. In this framework, one has a massive propagator  $(P^2 - m^2)^{-1}$ , and an additional interaction vertex proportional to  $m^2\phi^2$ . Of course, since this method adds and subtracts a mass term, exact results do not depend on the parameter  $m$ . In practice, since one truncates the perturbative expansion at a given finite order, there is a residual dependence upon  $m$  in the

result. Therefore, one has to choose the value of this mass. Some possibilities are listed below:

- (1) One can use the perturbative value for the thermal mass in this theory. At 1-loop, that amounts to choose  $m^2 = g^2 T^2 / 24$ , and this choice is equivalent to the resummation of HTLs.
- (2) One can get this mass from a gap equation.
- (3) Another possibility is to argue that since this mass term is not a physical quantity, one should choose  $m$  in order to minimize the sensitivity of the result on  $m$ .

These three methods give the same  $m$  at small  $g$ , but may differ for larger values of the coupling constant. The authors of [7] found that this reorganization of the perturbative expansion lead to significant improvements of its convergence, as illustrated on Fig. 4.



**Figure 4.** Results of the screened perturbative expansion for the free energy as a function of the coupling constant in scalar field theory [7]. (a) and (b): first two orders in screened perturbative expansion. (c) and (d): first two orders in the bare perturbative expansion [6].

The idea that it may be enough to tune the mass of quasi-particles to obtain a good estimate of the thermodynamical properties of the system has also been explored for QCD in [8]. The authors of this work show that it is possible to reproduce the lattice results for the pressure of a  $SU(3)$  Yang-Mills gas just by introducing a mass in the propagator of gluons. Physically, the fact that the pressure goes down when  $T \rightarrow T_c^+$  is a sign that partons feel a strong attractive force, and can be seen as a precursor sign of confinement. In order to reproduce this behavior, the mass introduced in [8] has to increase when  $T \rightarrow T_c^+$ , mimicking the fact that partons are trapped in heavy bound states.

### 2.3 Consistent approximation schemes

In addition to the fact that the applicability of the above ansatz to gauge theories is not obvious (it cannot be applied at higher orders in QCD because it lacks vertex corrections for gauge invariance), it also suffers from the fact that there are many ways to choose the mass parameter. A generalization of this method that overcomes the second of those problems can be derived by making use of a formula derived by Luttinger and Ward [9] for the thermodynamical potential. In the case of a scalar theory, this formula reads [10]:

$$\Omega = \frac{1}{2} TV \oint \{ \ln(-\Delta^{-1}) + \Delta \Pi \} + \Omega', \quad \Omega' = - \sum_n \frac{1}{4n} TV \oint \Delta \Pi_n, \quad (2)$$

where  $\Delta$  is the propagator,  $\Pi \equiv \Delta_o^{-1} - \Delta^{-1}$  the self-energy, and where the sum in the second line is a skeleton expansion containing only two particle irreducible vacuum contributions. This formula must be supplemented by the following variational principle:

$$\delta\Omega/\delta\Delta = 0, \quad (3)$$

which states that the thermodynamical potential should be stationary. This condition ensures thermodynamic consistency, and gives a relation between the self-energy and the propagator by  $TV\Pi = -2\delta\Omega'/\delta\Delta$ . Within this framework, one can derive approximation schemes that preserve consistency. Indeed, one can truncate the skeleton expansion of  $\Omega'$ , apply Eq. (3) to relate the propagator and the self-energy, and then compute the resulting  $\Omega$ . For instance, if one takes the following approximation for  $\Omega'$ :

$$\Omega'_1 = -\frac{TV}{4} \text{ (two circles) }, \quad \text{then one gets : } \Pi = \text{ (loop on a line) }. \quad (4)$$

Therefore, at this level of approximation, this ansatz is equivalent to the screened perturbation theory where the mass  $m^2$  is determined by a gap equation. As a consequence, results obtained at this level of approximation [10] are very similar to those displayed in Fig. 4.

At higher orders in the skeleton expansion, one would obtain non-constant self-energies with a much richer analytic structure: keeping higher orders for  $\Omega'$  has the effect to enlarge the functional space in which the minimum of  $\Omega$  is searched and therefore increases the chances to find a better minimum, closer to the absolute minimum that gives the exact  $\Omega$ .

This approximation scheme can in principle be generalized to gauge theories, where  $\Omega$  should be minimized with respect to the 2-point function, but also with respect to variations of vertices [11] (in order to preserve gauge invariance). Although possible in principle, this extended variational principle is very difficult to use due to its complexity. Instead of that, attempts have been made in which one trades the consistency for gauge invariance [12]: the gauge invariance is recovered by using the HTL approximation for propagators and self-energies, but the consistency is spoiled because HTLs are not exact solutions of the consistency equation.

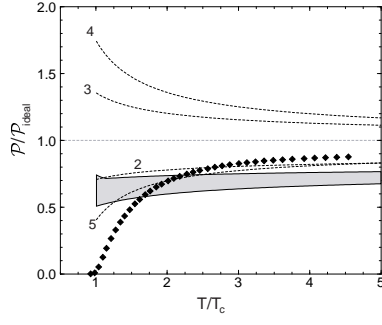
#### 2.4 HTL perturbative expansion

The spirit of this method [13] is very similar to the screened perturbation theory, applied to the case of QCD. One adds and subtract to the QCD lagrangian the term  $m^2\mathcal{L}_{HTL}$  that generates hard thermal loops:

$$\mathcal{L} = \mathcal{L} + m^2\mathcal{L}_{HTL} - m^2\mathcal{L}_{HTL}, \quad (5)$$

where I have made explicit the fact that the HTL lagrangian is proportional to a thermal mass  $m^2$ . The subtracted part is treated as counterterms. Again, since the full Lagrangian does not depend on  $m^2$ , one is free to choose  $m^2$  at will. One possibility is to use the perturbative value  $m^2 \sim g^2T^2$  for this parameter [2]. At 1-loop order, this gives the result displayed in Fig. 5.

At higher orders, it is possible to determine  $m^2$  in order to minimize the free energy with respect to  $m^2$ . This is very similar to the variational principle discussed above, restricted to the 1-dimensional sub-manifold spanned by the HTL propagator. This restriction tremendously simplifies the variational principle, while maintaining exact gauge invariance (because  $\mathcal{L}_{HTL}$  is gauge invariant). Obviously, this method will give good results if (and only if) this 1-dimensional sub-manifold gets close to the absolute minimum. In other words, it will work if the HTLs include the relevant physics of the QGP thermodynamics...

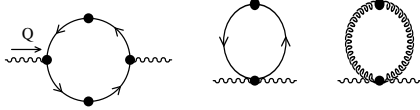


**Figure 5.** Results of the 1-loop HTL perturbative expansion (shaded area) for the normalized pressure [13]. Dotted curve: lattice data from [5]. 2,3,4,5: 2nd, 3rd, 4th and 5th order results in the bare perturbative expansion [3].

### 3. Photon and dilepton production

#### 3.1 1-loop calculation

Another physical quantity of interest for a quark gluon plasma is its photon production rate. Indeed, the size of the QGP expected in nuclei collisions is smaller than the photon mean free path (photons are weakly coupled). As a consequence, a photon produced in the plasma can escape and be detected without undergoing other interactions. Photons and dileptons are therefore very clean probes of the state of the matter at the time they were emitted. In the following, I focus mainly on photons with a small invariant mass, since this is the region where the calculation is the most sensitive to problems like infrared and collinear singularities.

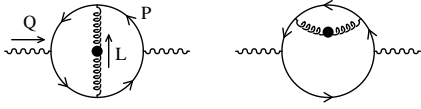


**Figure 6.** 1-loop diagrams for the photon polarization tensor in the HTL effective perturbative expansion.

In thermal field theory, the photon/dilepton rate is proportional to the imaginary part of the photon polarization tensor  $\text{Im} \Pi^\mu_\mu$  [14]. In the HTL effective theory, the 1-loop contributions to this 2-point function have been evaluated both for soft [15] and hard [16] real photons. The 1-loop diagrams are given in Fig. 6. For hard photons, one can neglect the HTL effective vertices, and the result was found to be of order  $\text{Im} \Pi^\mu_\mu(Q) \sim e^2 g^2 T^2$ . For soft photons, it was found to be of order  $\text{Im} \Pi^\mu_\mu(Q) \sim e^2 g^4 T^3 / q_0$ . In addition, the latter contribution has a collinear singularity that, after being regularized [17] by a thermal mass of order  $gT$ , gives an extra factor  $\ln(1/g)$ . For the contribution to soft photon production, one can note that the 1-loop result is extremely suppressed (it behaves like  $g^4$ ), because these 1-loop diagrams have a very small phase space since the loop is soft. Another problem with this 1-loop calculation (both for hard and soft photons) is that bremsstrahlung does not seem to contribute, contrary to results known from classical plasma physics.

#### 3.2 2-loop calculation

In fact, one can check that the above two problems are solved if one takes into account the 2-loop contributions of Fig. 7. Indeed, these contributions contain bremsstrahlung, and



**Figure 7.** 2-loop diagrams for the photon polarization tensor in the HTL effective perturbative expansion.

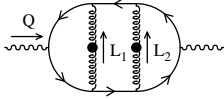
have a large phase space since the quark loop is hard. The latter property is enough to compensate the two additional  $q\bar{q}g$  vertices. In fact, it turns out that due to strong collinear singularities regularized by an asymptotic thermal of order  $gT$ , the 2-loop contribution is dominant, and of order  $\text{Im}\Pi^\mu_\mu(Q) \sim e^2 g^2 T^3/q_0$  for soft photons [18] and of order  $\text{Im}\Pi^\mu_\mu(Q) \sim e^2 g^2 q_0 T$  for hard photons [19]. In both cases, the numerical prefactor is of order 1 if  $Q^2/q_0^2$  is small, but decreases very fast when  $Q^2$  increases. In fact, one can check that the collinear enhancement is controlled by the following quantity [18]

$$M_{\text{eff}}^2 \equiv M_\infty^2 + \frac{Q^2}{q_0^2} p(p + q_0) , \quad (6)$$

where  $M_\infty \sim gT$  is the asymptotic thermal mass of a hard quark, and  $p$  is the momentum of the quark. The prefactor turns out to be of order  $g^2 T^2/M_{\text{eff}}^2$ .

### 3.3 Infrared divergences and KLN cancellations

Looking carefully at the 2-loop diagrams in Fig. 7, one can check that the kinematics prevents the gluon momentum  $L$  from becoming arbitrarily small. However, if one considers higher loop diagrams like the one depicted on Fig. 8, this the argument applies only to the sum  $L_1 + L_2$  of the two gluon momenta. Therefore, one gluon momentum can still go to



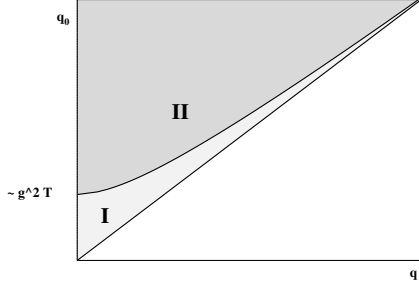
**Figure 8.** Example of 3-loop contribution to the photon polarization tensor.

zero and cause infrared divergences. One can estimate by power counting the order of magnitude of this 3-loop contribution [20], and one readily finds  $3\text{-loop} \sim 2\text{-loop} \times g^2 T/\mu$  where  $\mu$  is the infrared cutoff for the additional (space-like) gluon. If this extra gluon is longitudinal, then the cutoff is provided by the Debye mass of order  $gT$  so that the 3-loop contribution is suppressed by a power of  $g$ . But, if the extra gluon is transverse, then its cutoff can only come from the non-perturbative magnetic screening mass of order  $g^2 T$ , and therefore we have  $3\text{-loop} \sim 2\text{-loop}$ . This problem is very similar to the problem found by Linde for the free energy [21], but occurs in the leading order for photon production.

However, this power counting argument applies to individual cuts through the 3-loop diagram, but does not exclude IR cancellations when one sums the different cuts. In fact, it can be proven [20] that this sum over the cuts indeed leads to cancellations of infrared singularities, leaving a finite result without the need of a magnetic mass, a property which can be seen as a particular case of the Kinoshita-Lee-Nauenberg theorem [22]. More precisely, the sum over the cuts generates a kinematical cutoff of order  $l_{\text{min}} \sim q_0 M_{\text{eff}}^2/2p(p + q_0)$ , where  $M_{\text{eff}}$  is the mass introduced in the 2-loop calculation.

We are now left with two cutoffs that we must compare: the cutoff  $l_{\text{min}}$  that comes from the KLN theorem, and an hypothetical magnetic mass  $\mu$  at the scale  $g^2 T$ . Indeed, it may

happen that  $l_{\min}$  is smaller than the magnetic mass, and therefore does not prevent the result from being sensitive to the non-perturbative scale  $g^2T$ . This comparison has been done in



**Figure 9.** Comparison of  $l_{\min}$  and  $\mu \sim g^2T$  in the  $(q, q_0)$  plane. In region I, we have  $\mu > l_{\min}$  and the rate is sensitive to the magnetic scale despite the KLN cancellations: the photon rate is non-perturbative. In region II, we have  $\mu < l_{\min}$  so that we are not sensitive to the magnetic mass. In addition, in region II, we have 3-loop < 2-loop, and the photon rate can be treated perturbatively.

[20] and leads to a division of the photon phase space in two regions (see Fig. 9): for a large enough invariant mass (region II), the rate is insensitive to the scale  $g^2T$  and is dominated by the 2-loop contribution, while on the contrary low invariant mass photons (region I) are sensitive to the scale  $g^2T$  and one must resum an infinite series of ultra-soft corrections to estimate their rate.

### 3.4 Quark lifetime and LPM effect

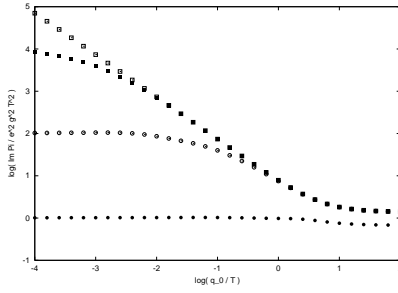
At the HTL level of approximation, the quarks have a thermal mass but their lifetime remains infinite. Indeed, their decay width  $\Gamma \sim g^2T$  appears only at the next order. A particular class of higher loop corrections which is worth studying by itself is the set of self-energy corrections that provides a width to the quarks. In fact, instead of adding perturbatively self-energy corrections on top of the 2-loop diagrams of Fig. 7, it is simpler to use a quark propagator already including a width. This amounts to perform the following transformation in the retarded propagator of the quark:  $(P^2 - M_\infty^2)^{-1} \rightarrow ((p_0 + i\Gamma)^2 - p^2 - M_\infty^2)^{-1}$ . The calculation with this propagator is rather straightforward, and leads to results that differ from the  $\Gamma = 0$  situation mainly by the expression of  $M_{\text{eff}}^2$ :

$$M_{\text{eff}}^2 = M_\infty^2 + \frac{Q^2}{q_0^2} p(p + q_0) + 4i \frac{\Gamma}{q_0} p(p + q_0) . \quad (7)$$

Since the width comes in the result via the ratio  $\Gamma p(p + q_0)/q_0$ , we see that it affects in an important way the rate of soft photons, while it modifies only marginally the rate of hard photons (if  $q_0 \rightarrow \infty$ , the imaginary part of  $M_{\text{eff}}^2$  is of the same order of magnitude as its real part). Numerical results for  $\text{Im} \Pi^\mu_\mu(Q)$  are displayed in Fig. 10, where it is easy to check the above remarks.

The next step is to determine the region of the photon phase space in which a width of order  $\Gamma \sim g^2T$  would be important. For that purpose, it is sufficient to compare the real and imaginary part of the complex number  $M_{\text{eff}}^2$ . They are of the same order of magnitude when  $2\Gamma \sim l_{\min}$ , where  $l_{\min}$  is the kinematical cutoff that appeared in the previous section. This makes obvious the fact that the region where the width is important is the same as the region where higher loop corrections are found to be sensitive to the scale  $g^2T$  (region I





**Figure 10.** Effect of the width on the bremsstrahlung as a function of  $q_0/T$  (for  $Q^2 = 0$ ). The various curves correspond to different values of the width  $\Gamma$ . From top to bottom, the ratio  $\Gamma T/M_\infty^2$  takes the values  $10^{-6}$ ,  $10^{-4}$ ,  $10^{-2}$  and 1.

of Fig. 9). This agreement is consistent with the fact that the width is dominated by the exchange of transverse gauge bosons of order  $g^2 T$  [23].

In addition, we can also give a much more physical interpretation for the non-perturbative region I. Indeed, since  $l_{\min}$  is the minimum momentum for an exchanged gluon, its inverse is the coherence length  $\lambda_{\text{coh}}$  for the emission (it can also be seen as the formation time of the photon). The inverse of  $\Gamma$  is the mean free path  $\lambda_{\text{mean}}$  of the quark in the plasma. The inequality  $l_{\min} < 2\Gamma$  defining region I can therefore be rewritten as  $\lambda_{\text{mean}} < \lambda_{\text{coh}}$ . This condition is nothing but the criterion for the Landau-Pomeranchuk-Migdal effect [24]. In fact, it is possible to rewrite the parameter  $M_{\text{eff}}^2$  that controls the bremsstrahlung process in a much more suggestive way:

$$q_0 M_{\text{eff}}^2 = 2p(p + q_0) \left[ \lambda_{\text{coh}}^{-1} + i\lambda_{\text{mean}}^{-1} \right]. \quad (8)$$

The LPM effect occurs when the formation time of the photon starts to be larger than the lifetime of the quark: successive scatterings of the quark cannot be considered individually, and multiple scatterings modify the production of the photon. In the language of thermal field theory, the photon production rate becomes sensitive to higher loop diagrams.

#### 4. Conclusions

Concerning the calculation of the QCD pressure, the first remark one can make is that the best results so far have been obtained in simple models that just give a mass to the gluons. Despite its lack of rigorous theoretical basis, this result indicates that most of the relevant physics can already be grabbed by having realistic quasiparticles. To include them more rigorously in the formalism, variational principles derived from the Luttinger-Ward formula seem to be a good option, but may be very complicated to implement for QCD. Simplifications can be obtained by restricting the variational space to the 1-dimensional sub-manifold spanned by HTL corrections, allowing one to have simultaneously realistic quasiparticles, thermodynamical consistency, and gauge invariance in a relatively compact formalism.

Concerning photon production by a QGP, it is now clear that the problem is non-perturbative if the invariant mass of the photon is too small. Indeed, the necessity of resumming higher order corrections appeared consistently both in the study of the infrared properties of higher loop diagrams, and when taking into account the finite timelife of the quarks. This breakdown of the perturbative expansion for low mass photons can be

interpreted as a manifestation of the LPM effect. However, finding a practical way to resum *all* the relevant contributions in this region is still an open question.

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